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REVIEWS OF BOOKS.

Logik der reinen Erkenntniss. Von HERMANN COHEN. Berlin, Bruno Cassirer, 1902. — pp. xvii, 520.

This volume forms the first part of a complete system of philosophy, and contains the fundamental principles by means of which the system itself is to be constructed. These principles arise in the sphere of pure knowledge; yet, while formal in origin, they serve to fashion their own content in the world of experience. They are not merely guiding, but they are determining principles as well. Such is Professor Cohen's main contention. In the development of his thesis he departs in many radical respects from the teachings of his master, Kant, whose interpreter he has been for many years, and in which office he has become widely known and appreciated. He has departed also in many essential particulars from his earlier work on *Kant's Theorie der Erfahrung*. That which is peculiarly characteristic of the present work is its insistence upon the essential purity of the thought processes, whose activity alone, it is maintained, produces their content. The following sentence, which I have chosen from many of a similar kind, will give an excellent idea of his general point of view: "Die verkehrte Ansicht dass das Denken, als Vereinigung, im Bilden von Ordnungen bestehe, hat ihren Grund in dem fundamentalen Vorurtheil, dass dem Denken sein Stoff von der Empfindung gegeben werde, und dass das Denken diesen Stoff nur zu bearbeiten habe. Dagegen denken wir auch die Mehrheit als zu erzeugende Einheit; auch für die Mehrheit die Aufgabe der Erzeugungs-Vereinigung. In dieser Bestimmtheit verstehen wir den Satz, dass die Thätigkeit den Inhalt erzeuge. Der ganze untheilbare Inhalt des Denkens muss Erzeugniss des Denkens sein. Und die ganze untheilbare Thätigkeit des Denkens selbst ist es welche den Inhalt bildet. Diese Einheit von Erzeugung und Erzeugniss fordert der Begriff des reinen Denkens" (p. 49).

From this general point of view, the author attempts to show how thought develops a system of fundamental judgments, which are the result of the pure processes of thought itself, and which function as determining moments in constructing the world of knowledge. These judgments follow in a general way the Kantian scheme of the categories. They are as follows:

1. Corresponding to the category of quality are the judgments expressing the laws of thought, which are divided into the judgments of (1) origin (*Der Ursprung*), (2) identity, and (3) contradiction.

2. Corresponding to the category of quantity are the judgments of mathematics which are divided into the judgments of (1) reality, (2) multiplicity, and (3) allness.

3. Corresponding to the category of relation are the judgments of applied mathematics, which are divided into judgments of (1) substance, (2) law, and (3) concept.

4. Corresponding to the category of modality are the judgments of method, which are divided into judgments of (1) possibility, (2) actuality, (3) necessity.

Such is the program of the logical foundations of the philosophic system. In this chain of the elements of pure thought, the primary link is found in the judgment of origin (*Ursprung*). It is the immediate conviction that every element of thought, every object of consciousness, must be traced to its first principle (*ἀρχή*). It is in the analysis of the implications of the primary principle that Professor Cohen finds the 'promise and potency' not only of the form, but of the stuff of all thought. For he discovers the common *Ursprung* of the varied forms of being in the infinitely small elements which constitute the ultimate parts of the world of reality. All that is finite has its origin in the infinitesimal. This is a matter of pure knowledge, because it rests upon the fundamental principles of mathematics as contained in the infinitesimal calculus. He contends, moreover, that the process of integration is one which is based essentially upon the principle of continuity, and inasmuch as the process from the infinitely small to the finite is a continuous one, there is in the unity thus established a fundamental basis of reality. Thus he says: "Die Continuität bedeutet daher den Zusammenhang der *dx*, den Zusammenhang der infinitesimalen Elemente. Man ist nicht mehr angewiesen auf einen anderer fraglichen Zusammenhang; nur dieser innerlichste und intimste wird gefordert. Nur er kann genügen; nur er ist durchschlagend; jede andere Art des Zusammenhangs wird entbehrlich. Alle sonstigen Zusammenhänge beruhen und bestehen in Vergleichen, die Sprünge machen und Lücken lassen. Die Continuität der infinitesimalen Elemente dagegen bedeutet den stetigen Zusammenhang, die Continuität der Realität" (p. 115).

Moreover, from the idea of continuity, he deduces by an alleged necessary implication the idea of number. With this quantitative basis of reality, he passes to the fundamental principles of the natural sciences which he reduces to the one ground principle of motion. "Bewegung ist der Grundbegriff der mathematischen Naturwissenschaft. Der moderne Begriff der Bewegung, wie Galilei ihn bestimmte, hatte die Beharrung zur Voraussetzung; aber diese ist und bleibt die Beharrung der Bewegung; sie ist nichts weniger und nichts mehr als ein Correlat. Ihre Voraussetzung ist eben die infinitesimale Realität. Und man wird sich endlich entschliessen müssen, das Sein, an welchem es der Substanz nun einmal gebricht, in jener zu begründen. Diesen Zusammenhalt von Substanz und Realität fordert der Gang der Wissenschaft, die in ihrem dunkeln Drange des rechten Weges sich wohl nicht immer bewusst ist, nichtsdestoweniger aber so sicher ihn geht, als er ihre Geschichte vollzieht" (p. 502).

The concepts substance, energy, force, are to be regarded as the various manifestations of law; and this law is essentially that of motion. Moreover, the idea of law must be completed by that of end or adaptation which affords an *a priori* foundation for the biological sciences. Finally, this round of derived notions ends in the threefold methodology which follows the lines of the traditional division of the modal judgment. Professor Cohen, however, gives a peculiar interpretation to these judgments of possibility, actuality, and necessity. In the judgment of possibility there is found the incentive to research, the suggestion of hypothesis, and the beginnings of all speculation. In the judgment of actuality the real is determined under the conditions of space and time, and always in terms of magnitude. In the judgment of necessity is found the ground of all universal judgments and of their combinations in the syllogism.

Such, in brief, is Professor Cohen's account of the deduction of the various constructive principles of pure knowledge from the primary category of the *Ursprung.* Together they form the logical foundations of his philosophical system. The main question, however, which suggests itself is this: Are the foundations firmly grounded? If not, the superstructure must fall of its own weight.

The central principle of the entire system is the mathematical doctrine of the infinitesimal calculus, by means of which Professor Cohen endeavors to establish a continuous process from non-being to being through the integration of infinitely small elements. Such a process he regards as the primary warrant of all reality. It is a process, moreover, which occurs in pure thought alone, because the infinitesimal cannot be an object of perception, nor can it be represented by the imagination. Thus the author insists that "Das Urtheil des Ursprungs besagt nur, dass das reine Denken mit dem Ursprung beginnen müsse,

sofern es das Denken der Erkenntniss, also des Seins ist. Jetzt aber sehen wir, wie auf Grund des Ursprungs das Sein als Realität zur Definition gelangt. Das Unendlichkleine stellt es dar. Nur das Unendlichkleine vermag es. Und das Unendlichkleine kann es vollständig zur Vertretung bringen. Es giebt kein anderes Mittel, und es braucht kein anderes Mittel zu geben. Es ist nur sensualistisches Missverständniss des Unendlichkleinen wenn man nach einem anderen Mittel der Realität verlangt; wenn man im Besitze der Infinitesimal-Rechnung ein Mittel der Realität vermisst" (p. 113).

This position, however, which is central to the whole system, cannot be maintained in the light of modern mathematics. The mathematical theory of the calculus is not based upon the doctrine of infinitesimals, as Professor Cohen assumes. On the contrary, the Leibnizian theory of infinitesimals has been discarded, and the doctrine of limits has taken its place; moreover, in the doctrine of limits the idea of the infinitesimal has no place whatsoever. In support of this position, I quote the following from Russell's *The Principles of Mathematics:* "The infinitesimal calculus is the traditional name for the differential and integral calculus together, and as such I have retained it; although, as we shall shortly see, there is no allusion to, or implication of, the infinitesimal in any part of this branch of mathematics" (Vol. I, p. 325). Again: "In his (Leibniz's) first published account of the calculus, he defined the differential coefficient by means of the tangent to a curve. And by his emphasis on the infinitesimal he gave a wrong direction to speculation as to the calculus, which misled all mathematicians before Weierstrass (with the exception, perhaps, of De Morgan), and all philosophers down to the present day. It is only in the last thirty or forty years that mathematicians have provided the requisite mathematical foundations for a philosophy of the calculus; and these foundations, as is natural, are as yet little known among philosophers except in France" (p. 326). The latter reference is particularly to Couturat's *De l'infini mathématique.* Again, Russell says: "It is the doctrine of limits that underlies the calculus, and not any pretended use of the infinitesimal" (p. 329).

Moreover, according to the doctrine of limits, the differential cannot possibly be regarded as an intensive element of reality which is by nature essentially infinitesimal, but whose summation will give a finite magnitude. The differential has only a relative value in the mathematical process; it is a symbol or index of the limit. The gap between it and the limit is never bridged. It only indicates the limit, and is never transformed into it. It is in no sense an intensively real element of a finite continuum. To establish this point one may well cite the following quotation from Professor Peano: "From the fact that the infinitesimal segment cannot be rendered finite by means of any actually infinite multiplication, I conclude with Cantor that it cannot be an element in finite magnitudes" (Peano, *Rivista di mathematica*, Vol. II, p. 62). Also on this same point, the following from Mr. Russell: "The limit does not belong to the series which it limits; and in the definition of the derivative and definite integral we have merely another instance of this fact. The so-called infinitesimal calculus, therefore, has nothing to do with the infinitesimal, and has only indirectly to do with the infinite — its connection with the infinite being that it involves limits, and only infinite series have limits." These quotations will suffice to show Professor Cohen's central position to be untenable. The infinitesimal as the *Ursprung* of all reality is a conception which has no place in modern mathematics. But this conception is the foundation of the so-called system of pure thought. That system, therefore, cannot stand. However excellent the details of the superstructure may be, and there are many excellent phases of the system as presented by Professor Cohen, nevertheless, this basal weakness renders the system as a system wholly worthless. The parts cannot be built together upon such a foundation.

But suppose Professor Cohen's fundamental postulate as regards the nature and function of infinitesimals be granted for sake of argument, would his system then be able to justify itself? I think not, and for the following reason: Many of Professor Cohen's alleged judgments of pure thought could never have been framed were it not for the empirical data out of which they have arisen. Therefore, with such a dependence, they can not be called elements of pure thought. To take one example which will serve to illustrate a general tendency observable throughout this work, Professor Cohen declares that Newton's Laws of Motion are essentially judgments of pure thought, and that they lie at the basis of the entire system of mathematical physics: "Die mathematische Naturwissenschaft ist die Wissenschaft von der Bewegung. Dieser Wissenschaft Newtons liegen die drei Principien zu Grunde, die Newton als Gezetze der Bewegung (*leges motus*) bezeichnet hat" (p. 219). It should be observed that this passage occurs as a part of the author's attempt to show that the primary laws of the mechanical world have their origin in the sphere of pure thought. It is well known that Galileo, who formulated the two first of these laws, and Newton, whose name is especially associated with the third, were close observers of nature, and that these laws represent the formulated interpretation of what they observed in experience, rather than categories of pure thought disclosed in the mind prior to all empirical suggestion. Indeed, the bare conception of laws of motion came to them primarily through the observed uniformities of nature. I take as testimony in point the following quotation from Thomson and Tait: "An axiom is a proposition, the truth of which must be admitted as soon as the terms in which it is expressed are clearly understood. But, as we shall show in our chapter on 'Experience,' physical axioms are axiomatic to those only who have sufficient knowledge of the action of physical causes to enable them to see their truth. Without further remark we shall give Newton's Three Laws; it being remembered that, as the properties of matter *might* have been such as to render a totally different set of laws axiomatic, these laws must be considered as resting on convictions drawn from observation and experiment, *not* on intuitive perception."[1]

Professor Cohen makes the radical mistake of regarding physics as a science of derived mathematics. It is essentially a science of applied mathematics, but not a science of derived mathematics; the empirical data cannot be separated from the so-called pure elements of thought without doing violence both to the form and the matter of natural phenomena.

It is impossible, owing to the limitations of the space allotted to me, to enter into a detailed criticism of this volume. Inasmuch as it purports to be the beginnings of a system of philosophy, I have endeavored to point out two particulars of structural defect which in my opinion imperil the system as a whole. My contention has been that the system rests upon a mathematical doctrine which is regarded by modern mathematicians as wholly unsound; and secondly, that the so-called elements of pure thought out of which the system itself is constructed disclose an obvious admixture of the stuff of experience.

JOHN GRIER HIBBEN.

PRINCETON UNIVERSITY.

Le personnalisme suivi d'une étude sur la perception externe et sur la force. Par CHARLES RENOUVIER. Paris, Félix Alcan, 1903. — pp. 537.

The scope of this work, the last M. Renouvier published before his death in September, 1903, is exceedingly broad. It aims at a demonstration of the central doctrine of the Person as the ultimate reality,

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[1] Thomson and Tait, *Treatise on Natural Philosophy*, Vol. I, Part I, p. 240.